A parametric transfer function model optimized by PSO

Frédéric Bardolle

BRGM (French Geological Survey), 5 rue de la Goélette, 86280 Saint-Benoit, France
f.bardolle@brgm.fr

Introduction

Transfer function based models are widely used in hydrological modeling, especially for rainfall-runoff simulation. This poster presents a new model associating three elementary transfer functions in series along four parallel branches. Each transfer function is a parametric probability density function with some physical meaning regarding hydrological transfers. Model inversion is performed using the so-called particle swarm optimization (PSO) technique. The systemic model and the PSO method are tested on five water catchments from all around the world.

Model architecture

The hydrological model presented here is based on a systemic approach establishing relationships between hydrological signals by means of transfer functions. It has a versatile architecture which can associate three elementary transfer functions in series along four parallel branches (Fig. 1).

Consequently, an output O(t) (e.g., river discharge) is the sum of up to four convolution products:

$$O(t) = I_1 * H_1^{eq}(t) + I_2 * H_2^{eq}(t) + I_3 * H_3^{eq}(t) + I_4 * H_4^{eq}(t)$$

where I_i (t) is the ith input (e.g., rainfall) and H_i^{eq} (t) is the ith equivalent transfer function and equals the convolution of three elementary transfer functions:

$$H_i^{eq}(t) = K_i.(H_i^1 * H_i^2 * H_i^3(t))$$

where K_i is a gain factor, and H_i^α is the αth transfer functions of the ith branch.

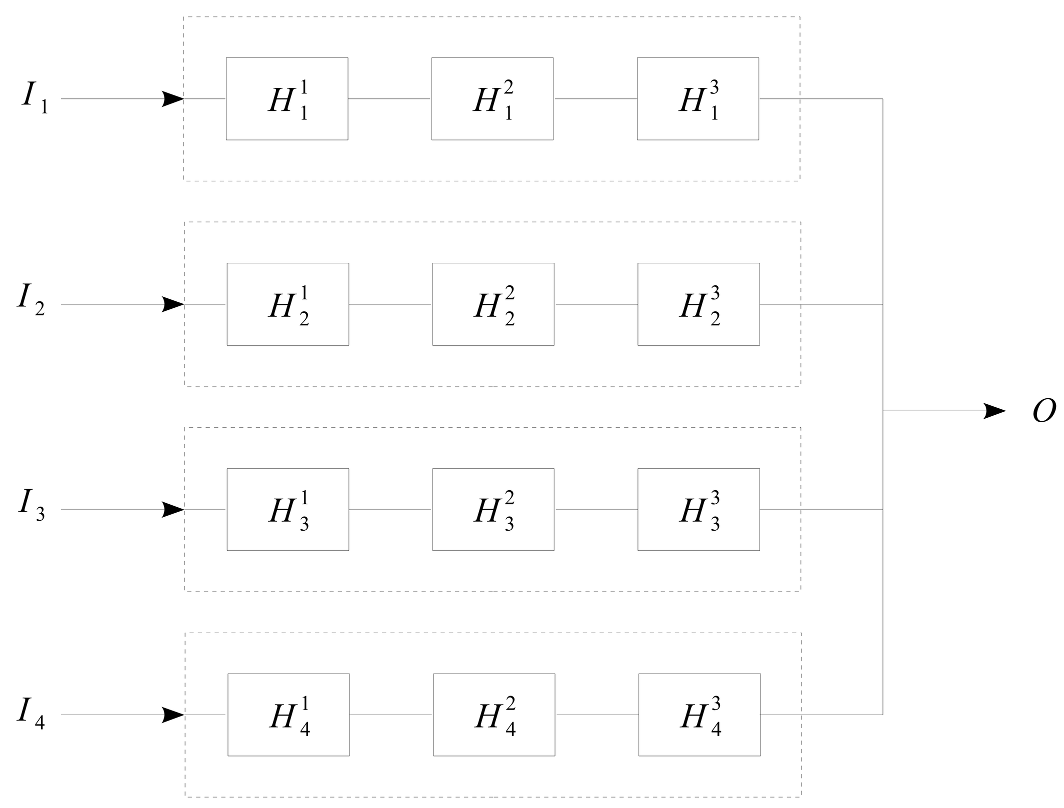

Fig.1 : Model architecture in series and parallel branches of transfer functions.

Regarding transient systems, parallel branches may correspond to multiple simultaneous transfer mechanisms or to some kind of spatial variability, whereas a series would link successive phenomena.

It is possible to choose the number of inputs (branches), and the number and type of elementary transfer functions H_i^α. These functions are selected from a library of parametric probability density functions with some physical meaning regarding hydrological transfers. For example, Normal distribution represents convection and dispersion effects and Power-law distribtution represents diffusion effects.

With one parameter per branch and two parameters per elementary transfer function, the model has between 3 and 28 parameters. These parameters are estimated with a swarm intelligence algorithm.

Inversion

Parameter estimation is performed by a particle swarm optimization (PSO) algorithm. Initially developed to simulate bird flocking, PSO consists in moving particles (vectors of size the number of parameters) in the parameter space to find the minimum of an objective function measuring a distance between simulated outputs and data (Fig. 2).

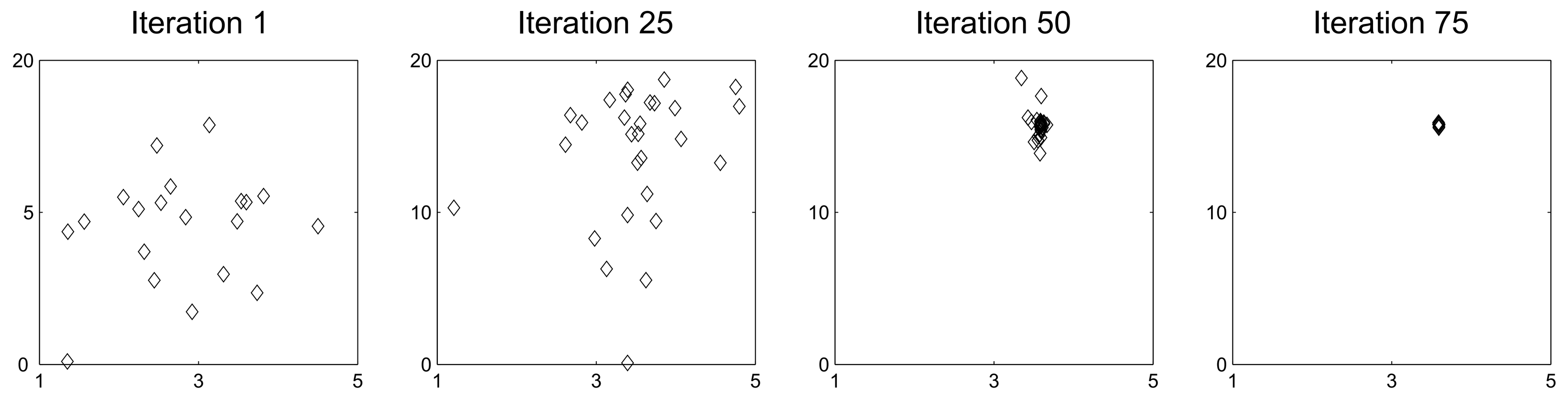

Fig. 2 : Projection of the position of a swarm of 30 particles, at iteration 1, 25, 50 and 75.

A particle i is moved iteratively and randomly in the parameter space while keeping for iteration t+1 the memory of its best experienced position p_i^t between [0,t] and the memory of the best experienced position among all the individuals of the swarm g^t. After an initialization step assigning to each particle a random location and a velocity in the parameter space, each iteration calculates the value of the objective function for each particle and updates velocities (v_i^{t+1}) and positions (x_i^{t+1}) as follow:

$$\begin{cases} v_i^{t+1} = w.v_i^t + c_1.r_1(g^t - x_i^t) + c_2.r_2(p_i^t - x_i^t) \\ x_i^{t+1} = x_i^t + v_i^{t+1} \end{cases}$$

where w is the inertia weight, c_1 and c_2 are acceleration constants, and r_1 and r_2 are random number uniformly distributed in (0,1).

PSO is well-suited to ill-posed inverse problems because its global search through the parameter space and the exchange of information between individuals lowers the risks of converging toward a local minimum.

Conclusion

Multi-input versatile architecture and parametric transfer functions optimized by PSO form a model that produces satisfying results for two of the five catchments.

For now, only the best solution is used but a stochastic model giving us an uncertainty measure of the simulation could be conceived with the other solutions found by the algorithm.

Results

The systemic model and the PSO have been faced to data of several river discharges: Blackberry Creek (USA), Durance (France), Gilbert (Australia), Kamp-Zwettl (Austria) and Lissbro (Sweden). All these water catchments experience non stationary conditions

The PSO seeks the parameters of transfer functions over five calibration periods P_i. The best inverse solution is then used to predict river discharge over the same five periods.

The input data are a daily effective rainfall rate, a daily potential evapotranspiration and sometimes an averaged daily air temperature over the basin (i.e., two to three branches in the model). The output data is a daily discharge. The Nash (N) criterion is calculated to compare precision of simulations for the various watershed mentioned above. It ranges between -∞ and 1, N=1 being the optimal value.

Fig. 3 : Value of Nash criteria for different calibration and evaluation periods, for five water catchment.

Nash criteria reported in Fig. 3 show that the model results in good simulations for both calibration and prediction periods for two catchments (Blackberry Creek and Lissbro). The model is accurate for calibrations of the Gilbert catchment but not for evaluations. Durance catchment and Kamp-Zwettl catchment simulations give poor results.